

# Quantum mechanics of superconducting nanowires

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In a short superconducting nanowire connected to bulk superconducting leads, quantum phase slips behave as a system of linearly (as opposed to logarithmically) interacting charges. This system maps onto quantum mechanics of a particle in a periodic potential. We show that, while the state with a high density of phase slips is not a true insulator (a consequence of Josephson tunneling between the leads), for a range of parameters it behaves as such down to unobservably small temperatures. We also show that quantum phase slips give rise to multiple branches (bands) in the energy-current relation and to an interband (“exciton”) mode.

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## I. INTRODUCTION

There is currently much interest in low-temperature properties of very thin superconducting wires. The key process, the significance of which one aims to understand, is a quantum phase slip (QPS),<sup>1</sup> a virtual depletion of the superconducting density through which the system tunnels to a different value of the supercurrent. The rate of this process depends not only on the “bare” fugacity—the tunneling exponential associated with the phase-slip core—but also on the interaction between individual QPSs.

The present study has been motivated by the observation<sup>2</sup> that, for a wire connected to bulk superconducting leads, as the length of the wire is reduced, the logarithmic interaction between QPSs crosses over to a linear one. Since we are talking about a tunneling process, the interaction takes place in the  $(x, \tau)$  plane, where  $x$  is the spatial coordinate and  $\tau$  is the Euclidean time. Heuristically, the change in the character of the interaction comes about because, during tunneling, the leads inhibit production of quasiparticles (plasmons) with nonzero wave numbers, so the “lines of force” in the  $(x, \tau)$  plane align in the  $\tau$  direction and form a “string” with a nonzero tension.

Now, it is well known that in the case of a logarithmic interaction, the system undergoes a phase transition that can be interpreted as unbinding of QPS–anti-QPS pairs. There are, in fact, several varieties of this phenomenon: a phase transition<sup>3</sup> in the Kondo problem and the equivalent transition<sup>4,5</sup> in the two-state dissipative quantum mechanics (DQM), the Berezinskii-Kosterlitz-Thouless (BKT) transition<sup>6–8</sup> in the XY model, and, finally, the superconductor-insulator transition (SIT) in the DQM with a periodic potential.<sup>9,10</sup> Long superconducting wires with significant amounts of disorder fall into the latter universality class.<sup>11</sup> Is there a similar transition in short wires where the interaction between QPSs is linear? Experiments do bring out a difference between short<sup>12,13</sup> and long<sup>14,15</sup> wires and suggest that a sharp transition exists in the former case.

Here, we present two theoretical results concerning this problem. First, we point out that in a short wire connecting bulk superconductors, there is no absolute distinction between a superconductor and an insulator: even an “insulator” can support a weak supercurrent due to Josephson tunneling. Second, we show that there is, nevertheless, a range of pa-

rameters for which the wire acts as an insulator down to unobservably small temperatures.

There is a mean-field-type approach<sup>16</sup> to short wires that models QPS as an effective resistive environment. In that approach, the renormalization-group equation for the QPS fugacity is taken to be the same as in a long wire, except that the plasmon impedance is replaced by an effective shunt resistance. The latter is determined self-consistently and includes the QPS channel ( $R_{ps}$ ), quasiparticles ( $R_{qp}$ ), and the impedance of the electrodes ( $R_{elec}$ ). While this procedure may qualitatively reflect the correct physics, it is not clear if it can be justified from first principles. Moreover, for the superconducting state in the limit when  $R_{elec}=0$  and  $R_{qp} \gg R_{ps}$ , it does not reproduce the exponential dependence of  $R_{ps}$  on the temperature found<sup>2</sup> by direct calculation.

Here, we consider the problem of SIT in short wires (with bulk superconducting leads) starting directly from the description in terms of a classical gas of particles with linear interactions in one ( $\tau$ ) dimension (Sec. II). We study this system in two different regimes (Secs. III and V), where two different approximations are possible. In Sec. IV we pause to describe two potentially observable effects predicted by our theory: a breakdown voltage, which, under certain conditions, provides a direct measure of the QPS fugacity, and a curious “exciton” mode. Prospects for detection of the breakdown voltage in experiment are discussed in Sec. VI. Sec. VII is a conclusion.

## II. ONE-DIMENSIONAL GAS WITH LINEAR INTERACTIONS

Our starting point is the partition sum of linearly interacting charges, which represent QPS and anti-QPS. This can be conveniently written as a path integral over an equivalent electrostatic potential  $\phi(\tau)$ ,

$$Z = \int \mathcal{D}\phi(\tau) \exp \left[ -\frac{1}{2K} \int d\tau (\partial_\tau \phi)^2 \right] \times \sum_{N_\pm=0}^{\infty} \frac{(\frac{1}{2}\alpha)^{N_++N_-}}{N_+! N_-!} \int \prod_{l=1}^{N_+} d\tau_l e^{i\phi(\tau_l)} \prod_{m=1}^{N_-} d\tau'_m e^{-i\phi(\tau'_m)}. \quad (1)$$

Note that the integral over the constant ( $\tau$ -independent) component of  $\phi$  enforces the condition of charge neutrality: only

terms with  $N_+ = N_-$  contribute to the sum. Most of the expressions below will be for zero temperature (i.e., infinite extent of the  $\tau$  dimension), but occasionally we will discuss consequences of the cutoff on  $\tau$  imposed by finite temperature.

The parameters appearing in Eq. (1) are  $\alpha$ , the bare QPS fugacity, and  $K$ , the strength of the linear interaction. The role of  $K$  can be understood by noting that, for a single QPS–anti-QPS pair, the path integral over  $\phi(\tau)$  would be

$$\int \mathcal{D}\phi(\tau) \exp \left[ -\frac{1}{2K} \int d\tau (\partial_\tau \phi)^2 + i\phi(\tau_1) - i\phi(\tau_2) \right] = e^{-(1/2)K|\tau_1 - \tau_2|}. \quad (2)$$

These two parameters encode a wealth of microscopic details of the phase-slip process. For example, if QPSs are rare,  $\alpha$  is proportional to  $\exp(-S_{\text{core}})$ , where  $S_{\text{core}}$ , the action of the QPS core, depends on the device specifics, such as the superconducting gap, the wire's width and thickness, and the width and thickness variations. When the latter are significant, QPS may favor a constriction, making  $\alpha$  independent of the wire's length  $L$ . On the other hand, in a uniform wire, QPSs occur preferentially at the center,<sup>2</sup> but the preference is rather weak, and  $\alpha$  scales linearly with  $L$ .

The value of  $K$  is determined by the one-dimensional superconducting stiffness  $K_s(x)$  (including possible variations of  $K_s$  with the width and thickness),

$$\frac{1}{K} = \frac{1}{4\pi^2} \int \frac{dx}{K_s(x)}. \quad (3)$$

Here  $x$  is the coordinate along the wire. For a uniform wire, this gives  $K \propto K_s/L$ , i.e.,  $K$  scales inversely with the length. The linear interaction in Eq. (2) then matches that obtained in Ref. 2. We recall that, from the microscopic point of view, the linear interaction between QPS reflects the energy of the initial and final tunneling states, in which fluctuations of the magnitude of the order parameter are small. This is why it is possible to find the strength of the interaction using the phase-only theory.<sup>2</sup>

In more detail, the relation of Eq. (1) to the phase-only theory is as follows. Consider the field  $\phi(x, \tau)$  dual to the phase  $\theta(x, \tau)$  of the superconducting order parameter in the sense that  $\partial_\tau \theta \propto \partial_x \phi$ . Since  $\partial_x \phi$  is proportional to the charge density,  $\phi$  can be thought of as the density of the electric dipole moment (in suitable units). When the leads are bulk superconductors, all  $\tau$ -dependent components of  $\theta$  satisfy the Dirichlet boundary conditions (b.c.) at the ends of the wire.<sup>2</sup> In view of the duality relation above, the Dirichlet b.c. for  $\theta$  imply the Neumann b.c. for  $\phi$  at both ends.

Then, if  $\alpha$  is much smaller than the charging energy of the wire and the temperature is low enough, only the spatially uniform mode of  $\phi(x, \tau)$  matters, so instead of a field we have a single quantum-mechanical degree of freedom,  $\phi(\tau)$ . Leaving aside overall constant factors, one can think of it as being the total dipole moment of the wire or, alternatively, the total charge transported through the wire since some fixed initial moment of time. It also coincides with the equivalent potential that appears in Eq. (1).

Equation (1) is an effective low-energy theory, which encodes the high-energy details in the values of its parameters,

and so is valid only up to some cutoff frequency  $\Lambda$ . Since the theory does not include fermionic quasiparticles, the cutoff is of the order of or smaller than the superconducting gap  $\Delta$ . Sometimes we will need a way to explicitly include such a cutoff in our calculations. One possibility, which we use in Sec. V, is to modify the kinetic term of  $\phi$  so as to suppress modes above a cutoff frequency. Another possibility, suitable for Monte Carlo simulations, is to discretize time. Cutoff dependence is discussed in Sec. VI. For now, we consider the case when both  $K$  and  $\alpha$  are much smaller than  $\Lambda$ , so the cutoff does not matter.

The sums in Eq. (1) can be evaluated by noting that the sum over  $N_+$  at fixed  $N = N_+ + N_-$  is Newton's binomial, and the remaining sum over  $N$  is an exponential (of  $\alpha \int d\tau \cos \phi$ ). Thus, Eq. (1) is equivalent to the theory with the Euclidean action,

$$S = \int d\tau \left\{ \frac{1}{2K} (\partial_\tau \phi)^2 - \alpha \cos \phi(\tau) \right\}. \quad (4)$$

This is similar to the mapping between the Coulomb gas and the sine-Gordon model in two dimensions.<sup>17–19</sup>

Equation (4) is quantum mechanics of a particle in a cosine potential. Solutions to the corresponding Schrödinger equation (the Mathieu functions) are known, and occasionally we will refer to their properties. For most of the time, however, we will consider various limits of Eq. (4), for which the recourse to the exact solution is not necessary.

Our system of charges with linear interactions is reminiscent of electrostatics in one dimension—an often used metaphor of quark confinement (see, for example, Ref. 20). As known in that context, depending on the dynamics of the charges, such a system can be in either a plasma or a confinement phase. The distinction lies in the large-distance behavior of a correlation function of *fractional* external charges. We therefore consider

$$C_q(\tau) = \langle \langle e^{iq\phi(\tau)} e^{-iq\phi(0)} \rangle \rangle, \quad (5)$$

with arbitrary  $q$ . Double brackets denote averaging with  $\exp(-S)$ . In a plasma state, the charges can screen any external charge, integer, or fractional. This corresponds to Eq. (5) approaching a constant value at large  $\tau$  for any  $q$ . In the confinement phase, *integer* external charges can still be screened, by new charges nucleating from the vacuum, but fractional charges cannot. As a result, correlator (5) with fractional  $q$  goes to zero at large  $\tau$ .

In the condensed-matter context, a plasma state, where the equivalent charges (the QPS) are unbound, corresponds to an insulator and the confinement phase to a superconductor. We now argue that, in the present case, there is no true insulator, i.e., Eq. (5) always goes to zero for fractional  $q$ .

### III. HIGH PHASE-SLIP DENSITY

We begin with the limit that is the best candidate for an insulator,

$$\alpha \gg K, \quad (6)$$

when QPSs are abundant. In this limit, the dominant paths are those where  $\phi(\tau)$  spends most of the time in the vicinity

of a minimum of the cosine potential but occasionally makes a transition to a neighboring minimum.

A natural way to describe this physics is the semiclassical approximation. To take into account small fluctuations of  $\phi$  near  $\phi=0$ , we expand the cosine in Eq. (4) in powers of  $\phi$  and treat the  $\phi^2$  term as a “mass” and the higher powers as an interaction. The Green’s function of  $\phi$  in the free massive theory is

$$G(\tau) = \frac{K}{2\omega_e} e^{-\omega_e|\tau|}, \quad (7)$$

where  $\omega_e$  is the characteristic frequency,

$$\omega_e^2 = K\alpha. \quad (8)$$

Using this Green’s function as a propagator for perturbation theory, one finds that the perturbative expansion is controlled by the small parameter  $K/\alpha$ , and the asymptotic of correlator (5) at large  $\tau$ , in perturbation theory, remains close to 1.

In addition to these perturbative fluctuations, however, there are large fluctuations (instantons) connecting different minima of the cosine. They are solitons and antisolitons of the sine-Gordon model, for example,

$$\phi_{\text{inst}}(\tau) = 4 \arctan e^{\omega_e \tau}.$$

Their action is

$$S_{\text{inst}} = 8(\alpha/K)^{1/2}. \quad (9)$$

These instantons are in a sense dual to the QPS, so when QPSs are numerous, instantons are rare. Under condition (6), they form a dilute gas with average density,

$$\bar{n} \sim \omega_e \sqrt{S_{\text{inst}}} e^{-S_{\text{inst}}}. \quad (10)$$

Each instanton contained in the interval  $(0, \tau)$  contributes  $\exp(2\pi iq)$  to correlator (5) and each anti-instanton  $\exp(-2\pi iq)$ . Statistics of these is that of an ideal gas, i.e., is given by the Poisson distribution  $P(N)$ . Hence,

$$C_q(\tau) \approx \sum_{N_+, N_-} P(N_+)P(N_-) e^{2\pi iq(N_+ - N_-)} = e^{-2\pi i[1 - \cos(2\pi q)]}. \quad (11)$$

We see that, due to the instantons, fractional charges are confined, with the string tension proportional to the instanton density. Note, however, that in practice  $\tau$  is cutoff by the inverse temperature, so the confinement of fractional charges (i.e., superconductivity) will not be observed until the temperature gets as low as  $T \sim \bar{n}$ , which in the present case is exponentially small.

The underlying physics becomes more transparent if one observes that  $\partial_t \phi / 2\pi$  is the electric current in units of  $2e$  (here  $t = -i\tau$  is the real time and  $e$  is the electron charge). Thus, each instanton transports  $2e$  of charge through the wire. A delocalized state of  $\phi$  [a Bloch wave of theory (4)] corresponds to a steady current. At small  $K/\alpha$ , the instanton rate is strongly suppressed, and for many purposes the system behaves as an insulator, but even then a weak steady current is possible by tunneling of charges through the wire (the Josephson effect).

Note also that, in a uniform wire, both  $K$  and  $\alpha$  depend on the wire’s length  $L$ ,  $\alpha$  being directly and  $K$  inversely proportional to it. Decreasing the length connects smoothly the “insulating” state of a nanowire to the superconducting state of a Josephson junction (JJ).

#### IV. EXCITON AND BREAKDOWN VOLTAGE

The spectrum of theory (4) has a band structure. In the path-integral formalism, the quasimomentum of these bands appears as a  $\vartheta$  angle associated with the instanton number

$$Q = \frac{1}{2\pi} \int d\tau \partial_\tau \phi.$$

Thus, we generalize Eq. (4) as follows:

$$S \rightarrow S + i\vartheta Q. \quad (12)$$

Each instanton now contributes an additional factor of  $e^{\pm i\vartheta}$ , and in the dilute-gas approximation the dependence of the partition sum on  $\vartheta$  is

$$\frac{Z(\vartheta)}{Z(0)} = \exp \left[ 2 \int d\tau \bar{n} (\cos \vartheta - 1) \right]. \quad (13)$$

The average current is obtained by differentiating  $\ln Z(\vartheta)$  with respect to  $\vartheta$  and equals

$$I = \frac{i}{2\pi} \langle \langle \partial_\tau \phi \rangle \rangle = 2\bar{n} \sin \vartheta, \quad (14)$$

where  $I$  is in units of  $2e$ . The maximal (critical) current is  $I_c = 2\bar{n}$ . Equation (14) is of the familiar Josephson form, so we conclude that the quasimomentum (or  $\vartheta$  angle) of the  $\phi$  description is precisely the relative phase  $\Delta\theta$  of the order parameter at the two ends of the wire.

The interpretation of  $\Delta\theta$  as quasimomentum is particularly apt given that in the absence of phase slips (i.e., for  $\alpha = 0$ )  $\Delta\theta$ , multiplied by the superfluid density, is the total momentum of the superfluid in the wire. One can say that phase slips turn momentum into quasimomentum. This is similar to how phase slips enforce periodicity with respect to the magnetic flux enclosed by a superconducting nanoring.<sup>21</sup>

Note, though, that  $\ln Z(\vartheta)$  gives only one band of the band structure, namely, the one corresponding to the ground state. In fact, there are additional bands, which can be obtained from the exact solution to the quantum-mechanical problem (4). They correspond to additional branches of energy as a function of the quasimomentum,  $E(\vartheta)$ , as shown in Fig. 1. One consequence of this is the existence of an interband excitation, which we term the exciton. At a fixed biasing current  $I$ , the exciton connects states for which the energy curves have equal slopes  $dE/d\vartheta = I$ . Since these states have different values of  $\vartheta$  (in particular, for  $I=0$ , the excited state is a  $\pi$  state), the transition produces a pulse of voltage.

The exciton is distinct from the usual Josephson plasmon, and the band structure described above is distinct from that described in Ref. 22. The plasmon can be included in the theory by supplying  $\vartheta$  with nonstatic modes and generalizing action (12) into

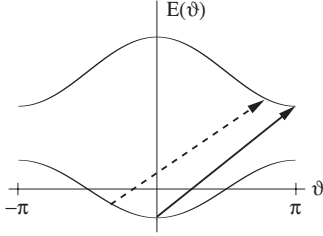


FIG. 1. A schematic of the band structure of theory (4). The arrows show exciton transitions for two different values of the biasing current  $I$ . Note that the exciton frequency in general depends on  $I$ .

$$\tilde{S} = S + \frac{i}{2\pi} \int d\tau \vartheta \partial_\tau \phi + T \sum_{n \neq 0} P(\Omega_n) |\vartheta_n|^2, \quad (15)$$

where  $T$  is the temperature,  $\Omega_n = 2\pi nT$  ( $n = \text{integer}$ ) are the Matsubara frequencies, and  $P(\Omega)$  is the plasmon response kernel. The interaction between the plasmon and the exciton, represented by the second term in Eq. (15), is the standard  $\hat{I}\vartheta$  interaction, with  $\hat{I} \propto \partial_\tau \phi$  describing the fluctuating current.

The only nonlinearity in Eq. (15) is the cosine potential in  $S$  [Eq. (4)]. If  $\vartheta(\tau)$  were the slowest variable in the problem, we could have, in the dilute instanton gas approximation, integrated  $\phi$  out, just as we did for a constant  $\vartheta$  in Eq. (13). This is possible if, at small  $\Omega$ ,  $P(\Omega)$  is purely capacitive,  $P(\Omega) \propto C\Omega^2$ , and the capacitance  $C$  is sufficiently small. In this case, we recover the theory of Ref. 22. We know, however, that, in general, superconducting leads modify the low-frequency response significantly. For effectively one-dimensional leads,  $P(\Omega) \propto |\Omega|/Z$  at small  $|\Omega|$ ,  $Z$  being the plasmon impedance of the leads (see Ref. 2), while for three-dimensional ones we expect  $P(\Omega)$  to go to a constant. The limit of “bulk” leads considered in this paper corresponds to  $P$  being large, so that integrating out the  $n \neq 0$  modes of  $\vartheta$  results only in a small correction to the kinetic term in Eq. (4). This allows us to neglect plasmons altogether in a study of the low-frequency (ground-state) properties. We note, however, that the plasmon-exciton coupling may become important when the device is operated at a higher frequency, e.g., by being subjected to microwave radiation.

Since  $\phi$  is proportional to the dipole moment of the wire, it couples, via a  $\phi V$  term, to the voltage  $V$  across the wire. For a wire deep in the “insulating” regime,  $\phi$  can sit for a long time near one minimum of the cosine potential. Its fluctuations there are small, and its value is related to the voltage by

$$V = \frac{\pi\alpha}{e} \sin \phi \equiv V_c \sin \phi. \quad (16)$$

At small  $V$ , the exciton is a transition to an excited state near the same minimum. The critical value  $V_c = \pi\alpha/e$  is the breakdown voltage, at which  $\phi$  begins to escape classically, and the system becomes a conductor.

Another breakdown channel, which opens at  $V > V_g \equiv 2\Delta/e$ , is production of quasiparticle pairs. However, the condition  $\alpha \ll \Lambda \lesssim \Delta$ , under which we have been operating

(and validity of which for realistic wires is discussed in Sec. VI), guarantees that  $V_c < V_g$ .

## V. LOW PHASE-SLIP DENSITY

We now consider the limit of small QPS fugacity,

$$\alpha \ll \min\{K, \Lambda\}, \quad (17)$$

where  $\Lambda$  is the ultraviolet (UV) cutoff. In this case, it is natural to expand the partition sum in powers of  $\alpha$ . This brings in correlators of the Gaussian theory,

$$S_0 = \frac{1}{2K} \int d\tau (\partial_\tau \phi)^2, \quad (18)$$

which need to be regulated in the infrared (IR). To this end, we modify Eq. (18) as follows:

$$S_0 \rightarrow \frac{1}{2} \int d\tau d\tau' \phi(\tau) M_0(\Lambda, \mu; \tau, \tau') \phi(\tau'), \quad (19)$$

where  $M_0$  is a differential operator whose inverse (the Green’s function)  $G_0 \equiv M_0^{-1}$  is

$$G_0(\Lambda, \mu; \tau, \tau') = \frac{K}{2} \left( \frac{1}{\mu} e^{-\mu|\tau-\tau'|} - \frac{1}{\Lambda} e^{-\Lambda|\tau-\tau'|} \right). \quad (20)$$

This implements an IR cutoff at frequencies of order  $\mu$  ( $\mu \ll \Lambda$ ). Indeed, for  $\mu|\tau-\tau'| \ll 1$ , Eq. (20) nearly coincides with the unregulated Green’s function, proportional to  $|\tau-\tau'| + \text{const}$ , but at large distances it decays exponentially, instead of growing linearly. In addition, Eq. (20) provides an explicit UV cutoff at frequencies of order  $\Lambda$ .

The requisite correlators are

$$\langle e^{iq\phi} \rangle = e^{-(1/2)q^2 G_0(0)}, \quad (21)$$

$$\langle e^{iq\phi(0)} e^{\mp i\phi(\tau)} \rangle = \exp \left[ -\frac{1}{2}(q^2 + 1)G_0(0) \pm qG_0(\tau) \right], \quad (22)$$

etc. Single angular brackets denote averages in the Gaussian theory, and we have used the shorthand notation  $G_0(\tau) \equiv G_0(\Lambda, \mu; \tau, 0)$ . We do not include a  $\vartheta$  angle in this section.

For simplicity, in this section we calculate, instead of correlator (5), directly its large  $\tau$  limit, i.e., the expectation value of the “disorder parameter,”

$$\langle \langle e^{iq\phi(0)} \rangle \rangle = \langle e^{iq\phi(0)} \rangle + \alpha \int d\tau \langle e^{iq\phi(0)} \cos \phi(\tau) \rangle_c + \dots, \quad (23)$$

where the subscript  $c$  denotes the connected part. The expectation value is even in  $q$ , and in what follows we assume  $q > 0$ .

The first term on the right-hand side of Eq. (23) rapidly vanishes as the IR cutoff is removed ( $\mu \rightarrow 0$ ), but the second one generally does not. The relevant piece is



$$\int d\tau \langle e^{iq\phi(0)} e^{-i\phi(\tau)} \rangle_c \approx \frac{4}{qK} e^{-(K/4\mu)(q-1)^2} f(K/\Lambda, q), \quad (24)$$

where  $f$  is a dimensionless function independent of  $\mu$ ;  $f(0, q) = 1$ . The approximation sign indicates that we have retained only the leading term in the  $\mu \rightarrow 0$  limit. We see that for any  $q \neq 1$  Eq. (24) vanishes exponentially at  $\mu \rightarrow 0$ , but for  $q = 1$  it remains constant.

The pattern repeats itself in higher orders in  $\alpha$ , except that now larger (integer) charges can also be screened. This is again the physics of confinement. Note that the main contribution to the integral in Eq. (24) for  $q = 1$  comes from small  $|\tau|$ , which means that the screening charge, represented by  $e^{-i\phi(\tau)}$  in Eq. (24), nucleates quite close to the external charge. For example, for  $K \ll \Lambda$ , the main contribution comes from  $|\tau| \sim 1/K$ . This is how it should be since, according to Eq. (2), the string tension is  $\frac{1}{2}K$ .

### VI. BREAKDOWN VOLTAGE IN EXPERIMENT

At temperatures  $T \ll \Delta$  and with enough disorder, the ratio  $K/\Delta$  depends on only one parameter—the total normal-state resistance of the wire  $R_N$ ,<sup>2</sup>

$$\frac{K}{\Delta} = \frac{2\pi^2 R_q}{R_N}. \quad (25)$$

Here  $R_q = \pi/2e^2 = 6.45 \text{ k}\Omega$  is the resistance quantum. So, for a wire with  $R_N \sim R_q$ , the ratio is  $K/\Delta \sim 20$ . Estimates suggest that the right quantity for numerical comparison with the UV cutoff  $\Lambda \lesssim \Delta$  is not  $K$  itself but  $\frac{1}{4}K$  or maybe even  $\frac{1}{8}K$ . In either case, though, such a wire is quite far from the cutoff-independent regime  $K, \alpha \ll \Lambda$ .

To locate the crossover between the “insulating” state and conventional superconductivity in the presence of a cutoff, we have used Monte Carlo simulations, with time discretized in steps of  $\Delta\tau = \pi/\Lambda$  and the crossover defined (somewhat arbitrarily) by the condition  $\langle \langle e^{i\phi} \rangle \rangle = 0.5$ . Preliminary data indicate that, at large  $K/\Lambda$ , the crossover occurs at a relatively small and nearly  $K$ -independent value  $\alpha_c \approx 0.4\Lambda$ . (Recall that  $\alpha$  is the fugacity per the entire wire and in a uniform wire is proportional to the length.) Therefore, we do not exclude that the observed zero-bias peaks in differential resistances of

“insulating” wires<sup>23,24</sup> are a manifestation of the electrical breakdown effect described by Eq. (16), and the excess (“offset”) voltage contained in such a peak provides an estimate of the breakdown voltage  $V_c$ .

### VII. CONCLUSION

In this paper, we have presented a theory of short superconducting wires that uses, as its basic variable, the wire’s electric dipole moment. We have found this description to be well suited to cases when the more conventional phase variable has been made “heavy” (i.e., its fluctuations have become inhibited) by sufficiently large, bulk superconducting leads. The prominence of the dipole moment (polarization) means that we view a small superconductor as if it was almost an insulator. We have seen, however, that, even in a thinnest wire, the true insulating behavior is prevented by the Josephson tunneling between the leads.

The thinner the wire is, the weaker that tunneling is. For sufficiently thin wires, it is described by instantons of theory (4), which are suppressed exponentially, so the insulating behavior persists down to exponentially low, unobservably small temperatures. The height of the potential traversed by the instantons,  $\alpha$  in Eq. (4), has the interpretation of the bare fugacity of quantum phase slips in the wire.

We have presented two potentially testable predictions of our theory. One effect, which, as discussed in Sec. VI, may already have been observed, is the existence of a breakdown voltage in “insulating” wires. The other is the exciton, corresponding to the interband transitions in theory (4). It can presumably be detected as a feature in the absorption spectrum. We should note, however, that Eq. (4) can be literally applied to computation of the exciton spectrum only when both  $\alpha$  and the exciton frequency are much smaller than the UV cutoff  $\Lambda$ .

*Note added in proof.* Recently, we learned about the work of Mooij and Nazarov.<sup>25</sup> They start from a somewhat different premise than we do, but their results overlap with ours.

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